MLTL Multi-type (MLTLM): A Logic for Reasoning About Signals of Different Types

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NASA Lunar Gateway: Assume-Guarantee Contracts[1]

[1] Dabney, James B., Julia M. Badger, and Pavan Rajagopal. "Adding a Verification View for an Autonomous Real-Time System Architecture." In AIAA Scitech 2021 Forum, p. 0566. 2021.

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$$(CMD == START) \rightarrow (\Diamond_{[0,5]}(ActionHappens \& \Diamond_{[0,2]}(CMD = END)))$$

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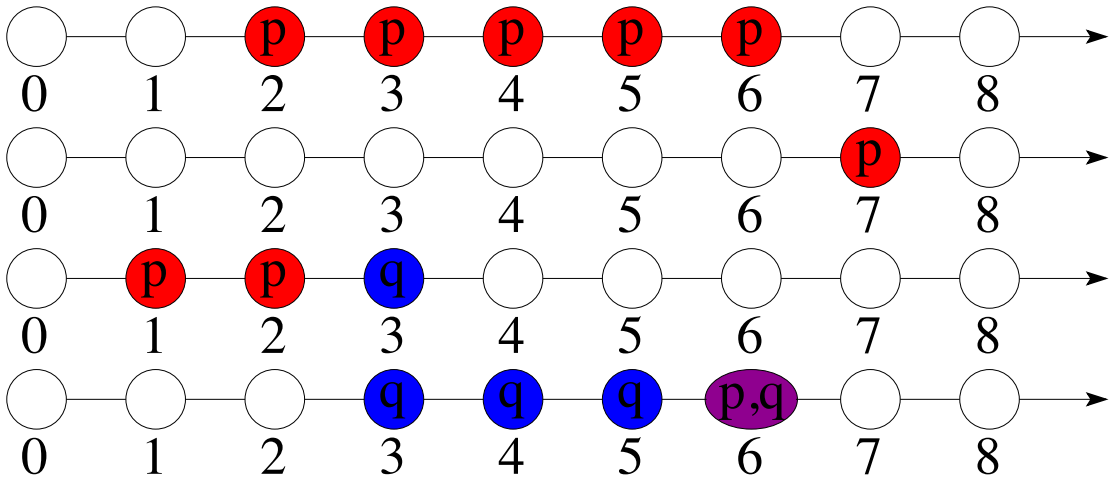
Encoding Finite Timelines

Mission-time LTL (MLTL) reasons about *bounded* timelines:

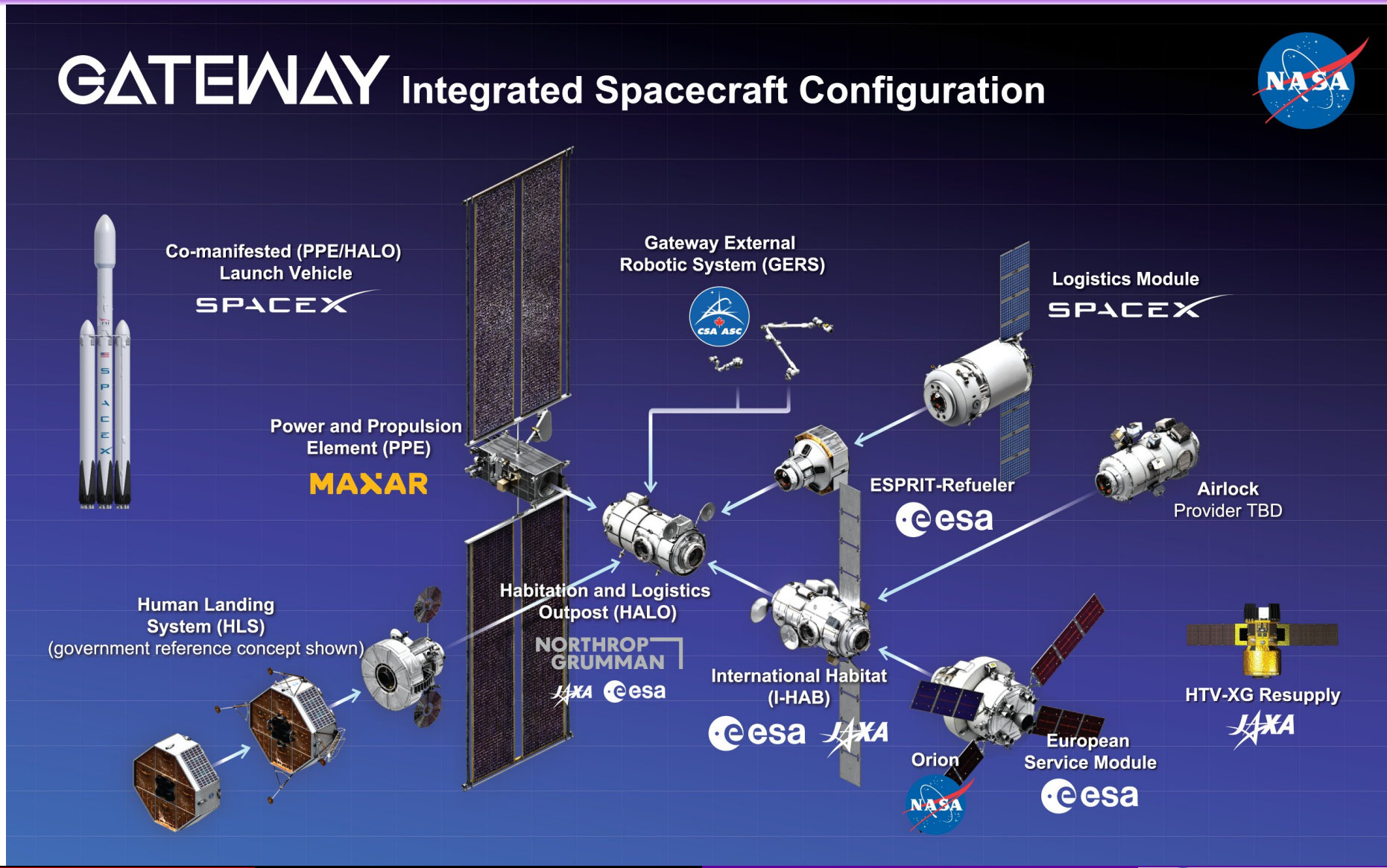
- finite set of atomic propositions {p q}
- Boolean connectives: ¬, ∧, ∨, and →
- temporal connectives *with time bounds*:

Symbol	Operator
$\Box_{[2,6]} p$	$\text{ALWAYS}_{[2,6]}$
$\Diamond_{[0,7]} p$	$\text{EVENTUALLY}_{[0,7]}$
$p \mathcal{U}_{[1,5]} q$	$\text{UNTIL}_{[1,5]}$
$p \mathcal{R}_{[3,8]} q$	$\text{RELEASE}_{[3,8]}$

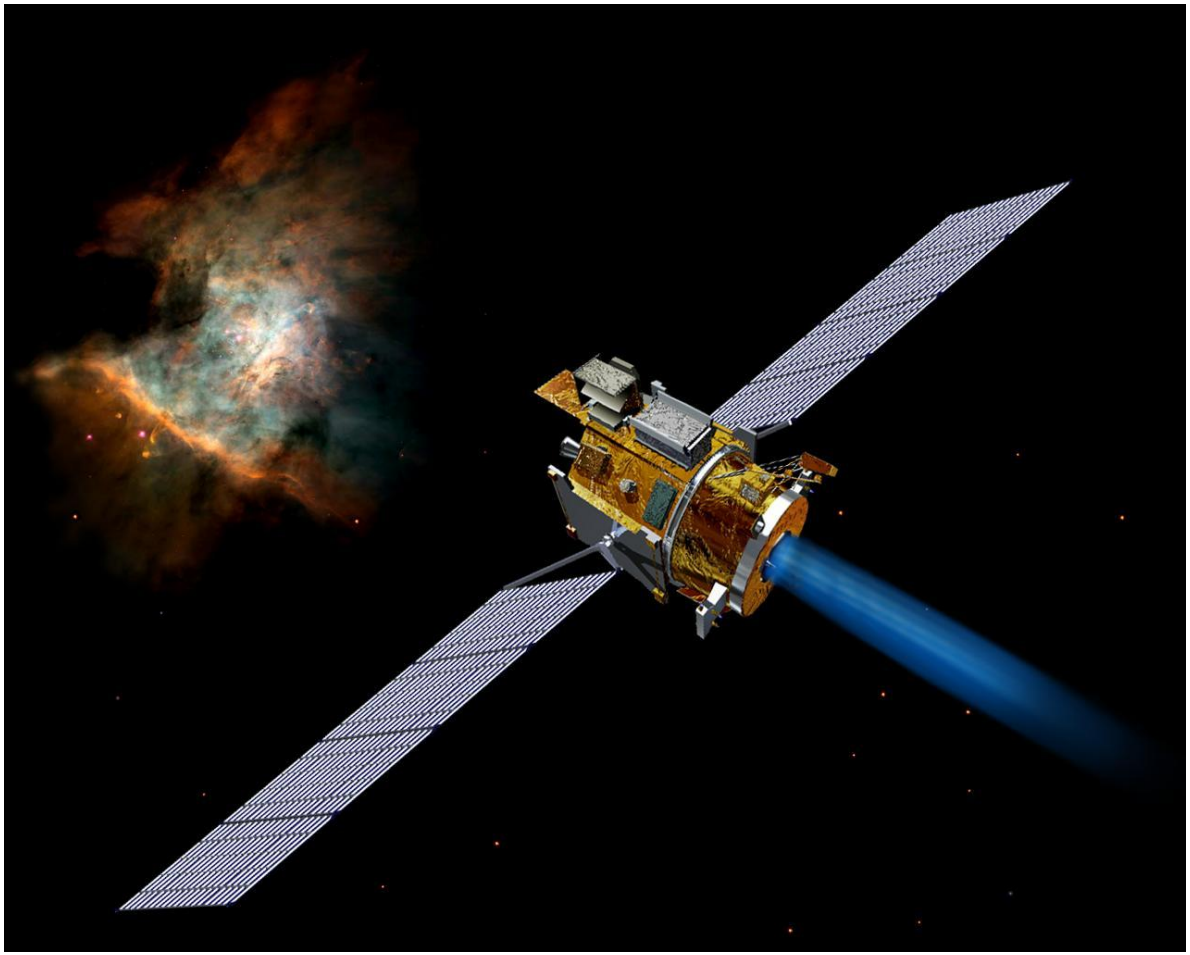


Mission-bounded LTL is an over-approximation for mission time τ

NASA Lunar Gateway: A System of (Mix-Typed) Systems!



A Typical Deep Space Mission



- *monthly* course corrections
- *nanosecond* precise sensor adjustments
- *system-level* requirements

The Question

Existing logics reason over signals of the *same type*:

$\pi = \{\sigma_0, \cdots, \sigma_n\}$ is a set of signals populating $p_0, \cdots, p_n \in \mathcal{AP}$

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What happens when signals have different types?

We Need A Logic For This!

Like MLTL but . . .

- includes type conversions different types in one formula

[2] K.Y.Rozier. "Specification: The Biggest Bottleneck in Formal Methods and Autonomy." VSTTE, 2016.

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We need MLTL *for mixed types*

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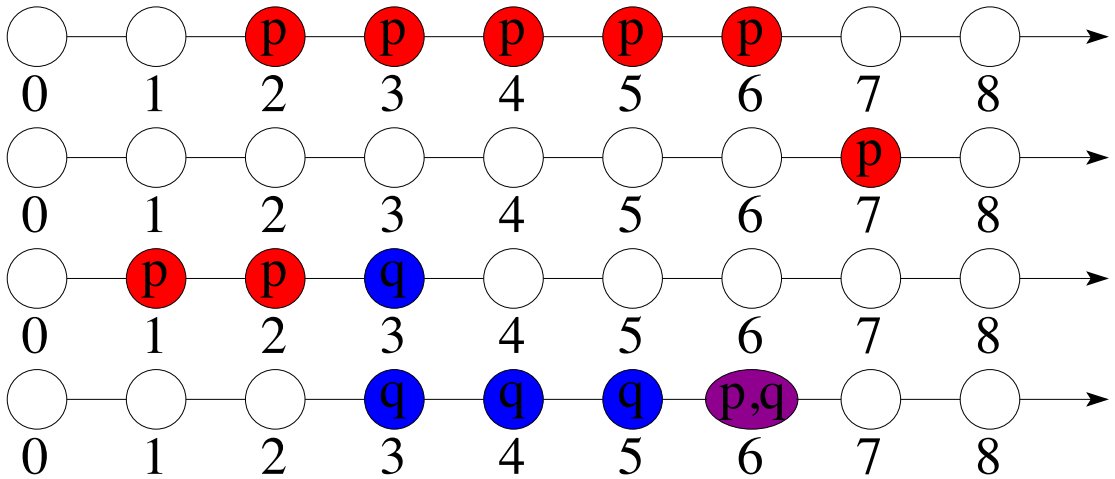
- **modularity**: clean separation of type conversions from MLTL structure
- **complexity**: fit in limited resources of embedded systems
- **validation**: use the right tool for the job, not a kludge
- **extensibility**: retain type conversions to enable optimization
  - store more information in one formula

Encoding Finite Trajectories Over Signals of Mixed Types

MLTL Multi-type (MLTLM) reasons about formulas over signals of *mixed types*:

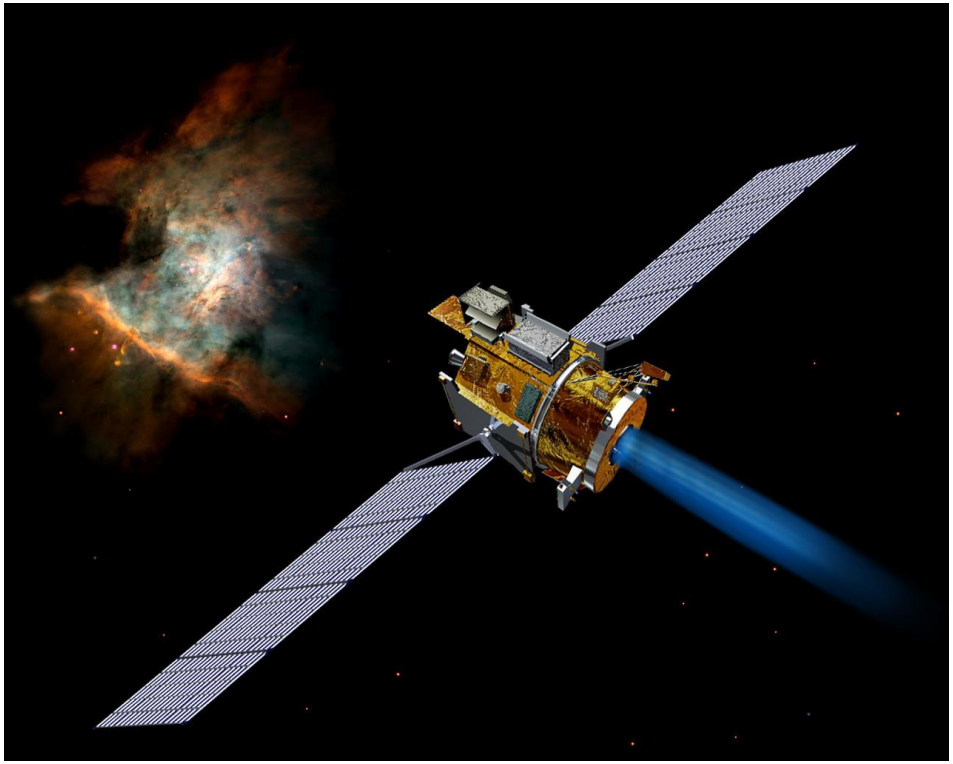
- finite set of atomic propositions {p q}
- Boolean connectives: $\neg$, $\wedge$, $\vee$, and $\rightarrow$
- temporal connectives *with time bounds*:

Symbol	Operator	Timeline
$\Box_{[2,6,\mathbb{A}]}p$	$\textsc{Always}_{[2,6,\mathbb{A}]}$	
$\Diamond_{[0,7,\mathbb{A}]}p$	$\textsc{Eventually}_{[0,7,\mathbb{A}]}$	
$p\mathcal{U}_{[1,5,\mathbb{A}]}q$	$\textsc{Until}_{[1,5,\mathbb{A}]}$	
$p\mathcal{R}_{[3,8,\mathbb{A}]}q$	$\textsc{Release}_{[3,8,\mathbb{A}]}$	



To resolve the formula, need a *projection*: $T^{\mathbb{B}}_{\mathbb{A}}(\sigma^{\mathbb{A}}) = \sigma^{\mathbb{B}}$

Example: Deep Space Mission

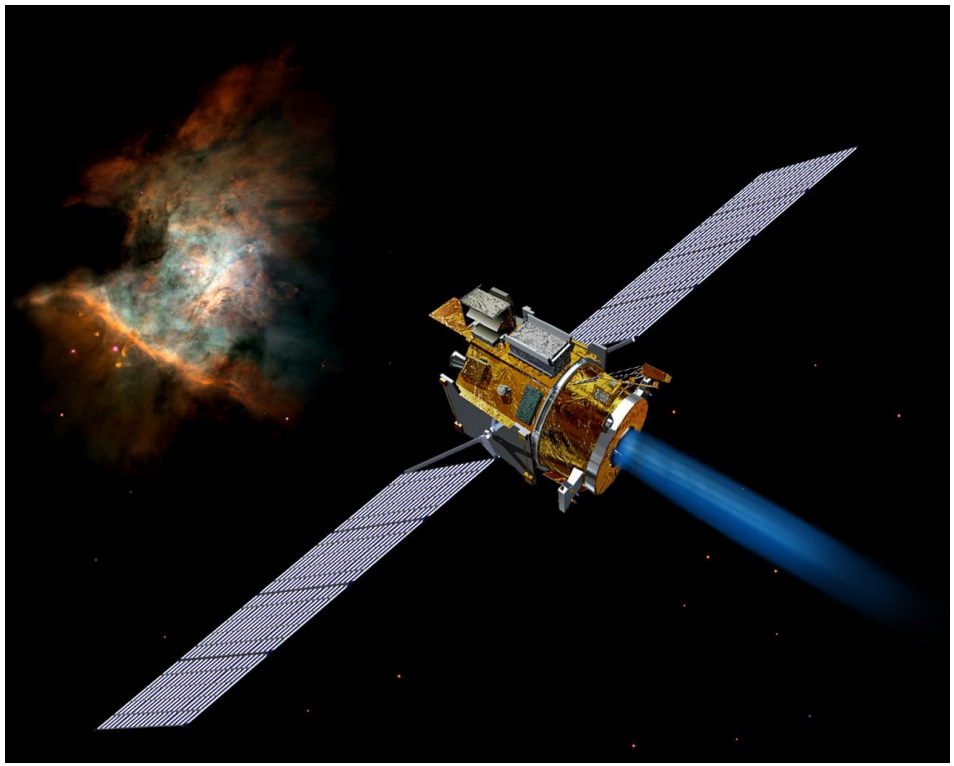


The spacecraft maintenance cycle runs at least once a month over the five-year mission.

Monthly course corrections never involve burning the thrusters more than 3 seconds at a time.

$$\square_{[0,5,year]}[(\Diamond_{[0,30,day]} maintenance) \wedge (\neg\square_{[0,3,sec]} thrusters)]$$

Example 2: Reference Frame Correction



Timing beacons from the Gateway and the spacecraft need to be in sync; typing accounts for relativity, clock drift, transmission delay (e.g., Lorenz translormation or simpler approximation)

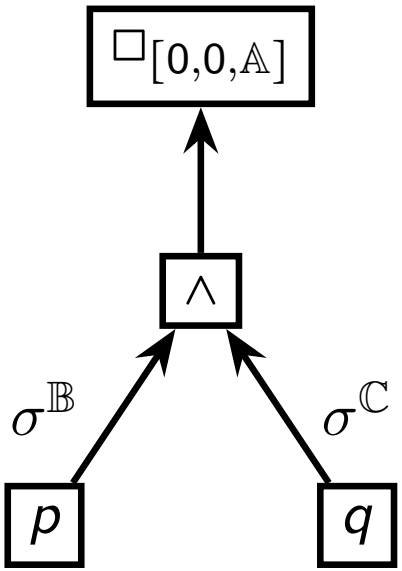
$$(\neg(BeaconSent_A \wedge G[0,10,A]BeaconSent_B)) \rightarrow \Diamond[0,1000,B]Resync$$

Relationship Between MLTL and MLTLM

Logical projection: a projection that can be expressed in MLTL

MLTLM with all logical projections is equivalent to MLTL

Projection Options & Implementation Patterns



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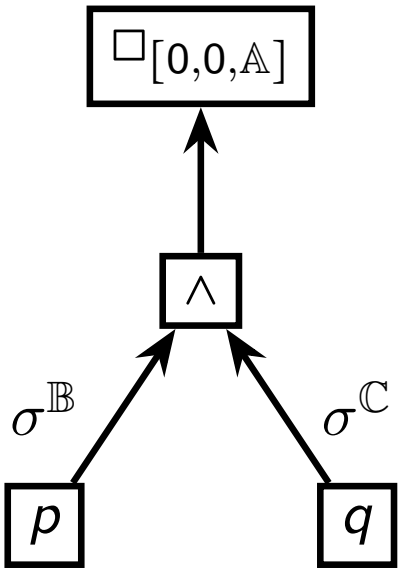

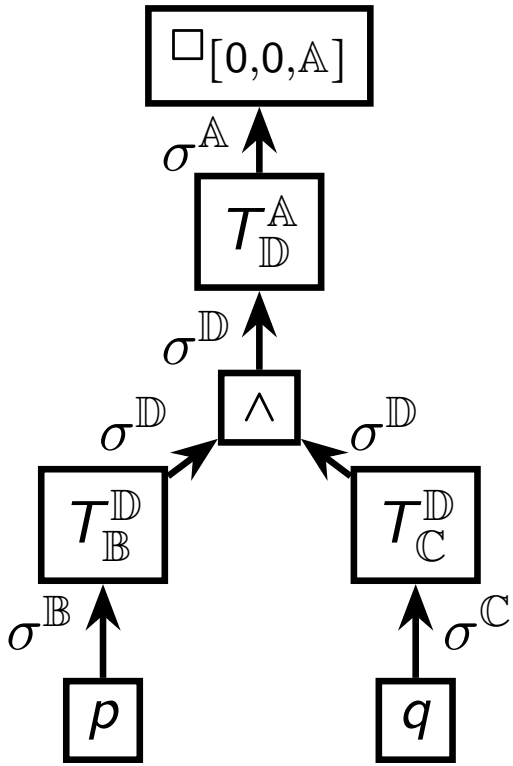

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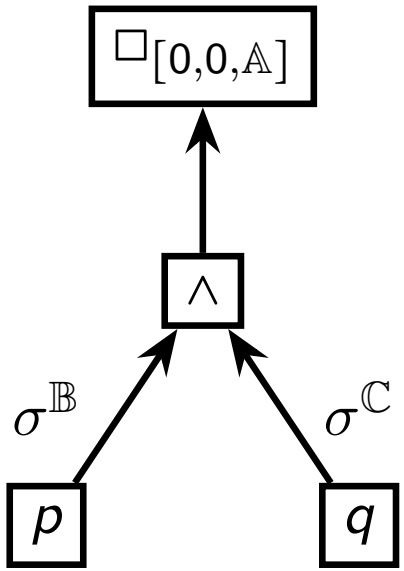
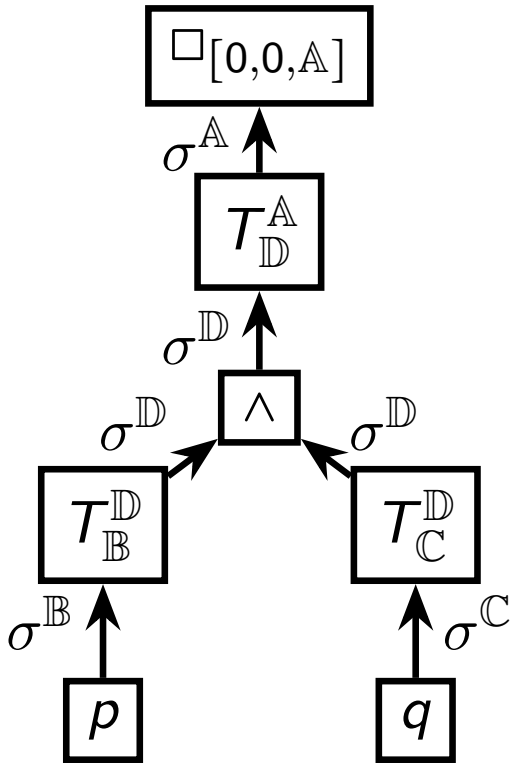
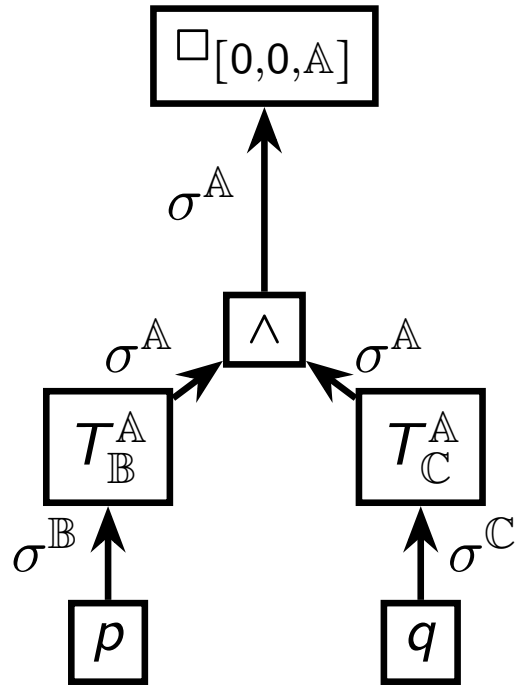
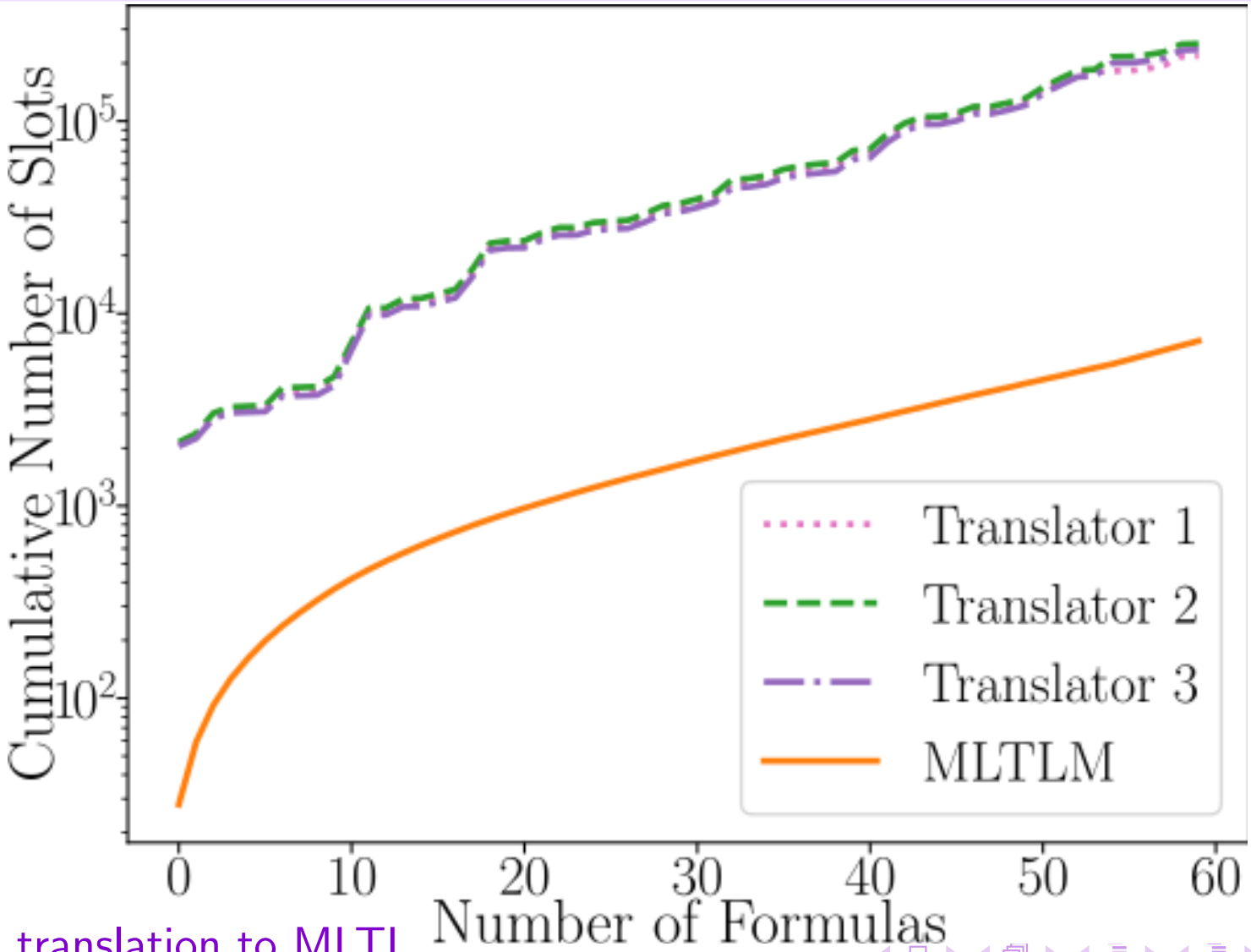




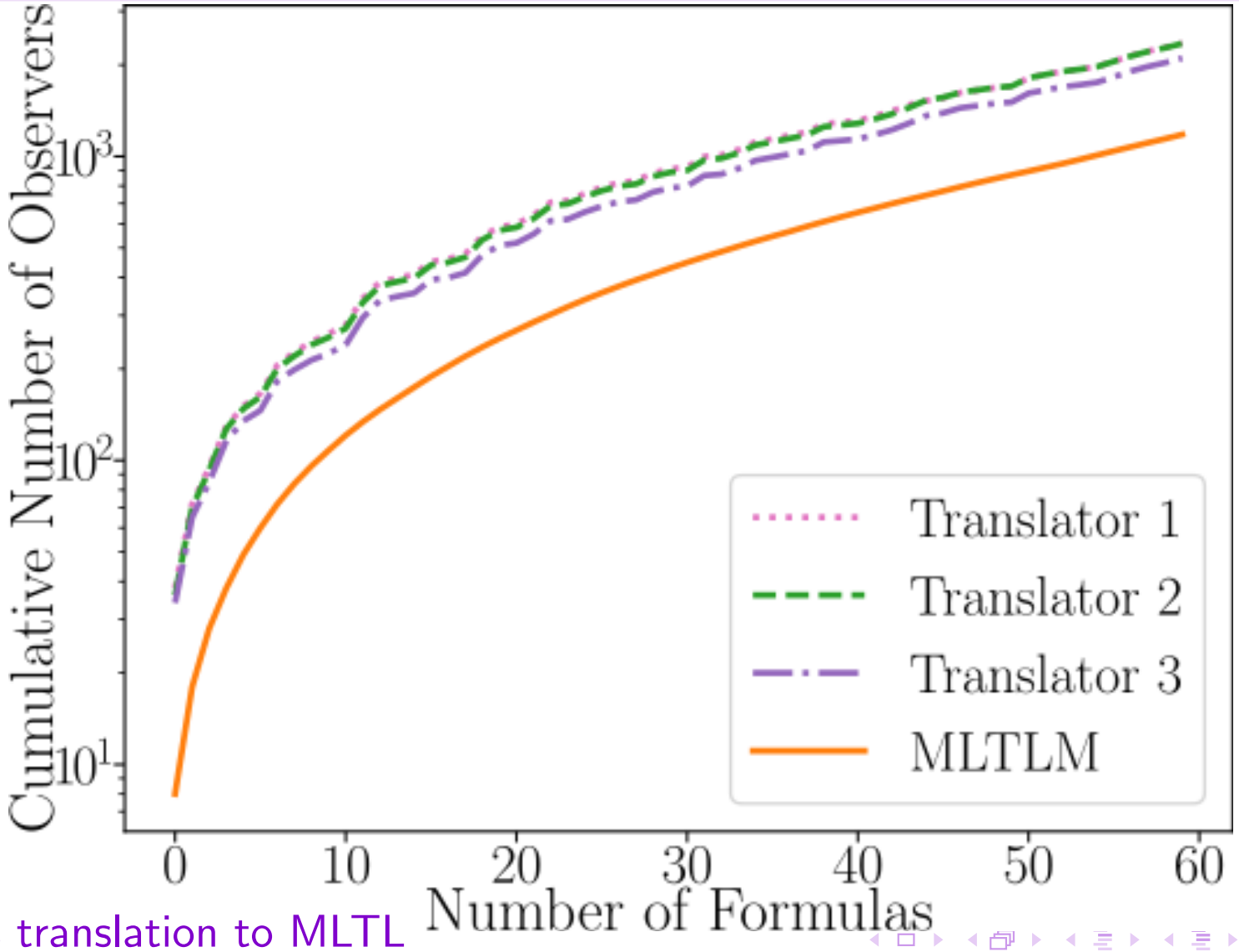


Direct Analysis of MLTLM Reduces Memory*



* versus translation to MLTL

Direct Analysis of MLTLM Reduces Time*



* versus translation to MLTL

Summary

- 3 translation algorithms: MLTLM w/logical projections → MLTL
- MLTLM RV algorithm & open-source implementation
- Direct MLTLM analysis saves space and time
- Preserve formula validation and modularity!

Mix your types with MLTLM!

http://temporallogic.org/research/NSV2022